

# Integrating Spatio-thematic Information

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**Abstract.** Mechanisms for overcoming semantic heterogeneity in diverse information sources are critical components of any interoperable system. In the case of diverse geographic information sources, such mechanisms present particular difficulties since the semantic structure of geographic information cannot be considered independently of its spatial structure. This paper develops an approach to the integration of semantically heterogeneous geographic information that is capable of addressing the spatial and thematic aspects of geographic information in concert. The approach is founded on an algebraic model of spatio-thematic information layers. Variations in the thematic structure of layers are explored, including the integration of layers with hierarchical thematic structure. The effects of changes in the thematic granularity of layers are also considered. Finally, a case study of the integration of heterogeneous land cover data is used to illustrate the relevance of the formal model within a practical application.

## 1 Introduction

For many organizations and applications, achieving interoperation between multiple information sources that have been developed in differing contexts for a variety of purposes is a vital capability. The issue of semantic heterogeneity of information sources has been a challenging problem since the early days of distributed database systems. Semantic heterogeneity has been investigated as part of all major information system architectures during the last three decades. The terms may have changed from “semantic mismatch” [1] and “semantic inconsistency” [2] to “merging ontologies” and “ontology alignment” [3], but the underlying problem of appropriately integrating information from different sources using different concepts, assumptions, and languages, remains. Indeed, with global inter-connectivity, the issue is more important and intractable now than at the outset of distributed information systems technology.

The problem of data fusion under conditions of semantic heterogeneity may be decomposed into *identification* and *resolution* [4]. The present work addresses both these questions in the context of geographic information sources. The process of identification enables us to select areas where semantic issues arise (often in the form of possible conflicts and inconsistencies). The process of resolution enables the fusion to take place, if possible, where inconsistencies can be worked

around or resolved. There is a growing literature on the ontological issues arising from semantic heterogeneity amongst geographic information sources (see, for example [5–7]).

We assume a structuring of the geographic information into *spatio-thematic layers*, where each information source associates a unique attribute value with each point in the space. Thus, a layer is similar to Berry and Tomlin’s sense of a map algebra [8, 9]. The novel aspect of this work is that we give each thematic space a structure, which might be hierarchical, or where themes are conceived of as regions having topological relationships in a theme space.

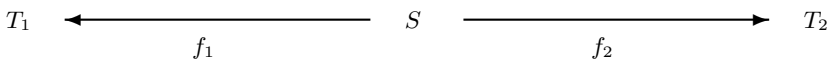
The approach will be constructed both formally and by means of small examples running through the formal development. Our major application has been to the domain of land cover data. Land cover data often possess rich thematic structure, such as taxonomies, in addition to their spatial structure. Attending to both spatial and thematic structure is important if the integration of heterogeneous land cover data is to be successful and meaningful. A specific example, discussed further in Section 4, addresses the integration of land cover data drawn from the European Co-ordination of Information on the Environment (CORINE) project and the Ordnance Survey of Great Britain (OSGB) Digital National Framework (DNF).

## 2 Layer Integration

Assume throughout that the underlying spatial framework is fixed and denoted  $S$ . Let  $T$  be a thematic space. Then a  $T$ -layer is formally a function  $f : S \rightarrow T$ . Integration of layers is expressed formally through the construction of a product layer. In general, the issue is well expressed by the configurations shown in Fig. 1 and 2. Suppose we have two layers, a  $T_1$ -layer  $f_1 : S \rightarrow T_1$  and a  $T_2$ -layer  $f_2 : S \rightarrow T_2$ , as shown in Fig. 1. The *product layer*  $f_1 \otimes f_2$  is defined as:

$$f_1 \otimes f_2 : S \rightarrow T_1 \otimes T_2$$

that makes the diagram in Fig. 2 commute. So, to construct a product layer, we need to construct the set  $T_1 \otimes T_2$ , function  $f_1 \otimes f_2$ , and the two projection functions  $p_1$  and  $p_2$ . In the general case, there are several ways to do this, and the specific way depends on the underlying structure given to the thematic spaces  $T_1$  and  $T_2$ . We now examine some of these cases in more detail.



**Fig. 1.** Two spatio-thematic information sources

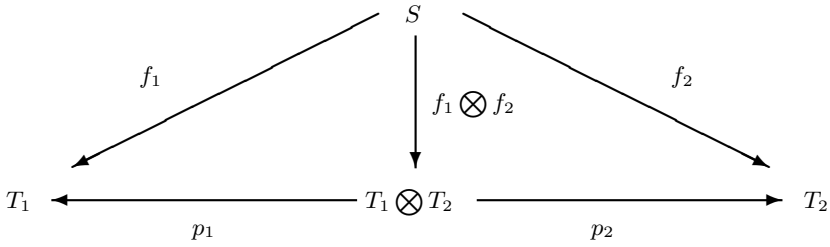


Fig. 2. Two spatio-thematic information sources and their product

**2.1 Case A: No Structure in the Thematic Spaces (Unconstrained Overlay)**

In this case, the usual Cartesian product construction as ordered pairs is appropriate. Assume given two layers,  $f_1 : S \rightarrow T_1$  and  $f_2 : S \rightarrow T_2$ , as shown in Fig. 1. Suppose that  $T_1$  and  $T_2$  are two thematic spaces with only an underlying set structure, so that each specific theme is just a member of  $T_1 \cup T_2$ . Then, the product construction gives the following:

$$T_1 \otimes T_2 = \{(t_1, t_2) : t_1 \in T_1, t_2 \in T_2\}$$

$$p_1 : (t_1, t_2) \mapsto t_1, \quad p_2 : (t_1, t_2) \mapsto t_2$$

$$f_1 \otimes f_2 : s \mapsto (sf_1, sf_2)$$

and it is not difficult to check that the arrow diagram in Fig. 2 commutes under this construction.

For example, consider the locational space  $S$  shown in Fig. 3. In set-theoretic terms,  $S = \{s_1, s_2, s_3, s_4\}$ . Suppose we have two thematic classifications, into nations USA ( $U$ ) and Canada ( $C$ ), and into land cover types heathland ( $H$ ) and woodland ( $W$ ). Then  $T_1 = \{U, C\}$  and  $T_2 = \{H, W\}$ . Then the integrated thematic space is given by:

$$T_1 \otimes T_2 = \{(U, H), (U, W), (C, H), (C, W)\}$$

and the layer is as shown in Fig. 4. Because of the way in which the layers are spatially structured, every element of  $T_1 \otimes T_2$  occurs in the product layer.

**2.2 Case B: Partition Structure in the Thematic Spaces**

Case A provides the simplest possible case, where the themes were all atomic and independent of each other. In this next case, themes are modeled as non-empty sets; blocks of a partition, rather than individuals. Assume that there is an underlying space  $U$  of “atomic” themes, and each of the thematic spaces  $T_1$  and  $T_2$  is a granulation of  $U$ . Note, that this assumption of a common underlying

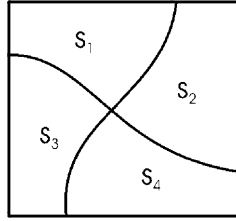


Fig. 3. Example partition of space

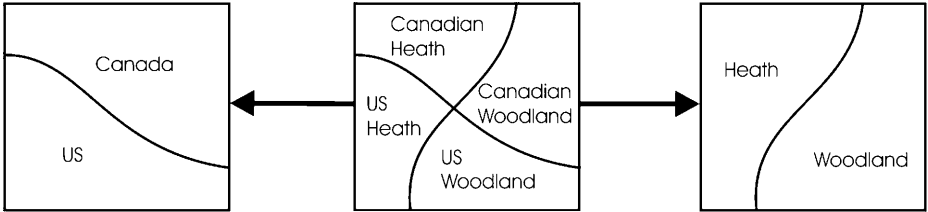


Fig. 4. The product of two layers with unstructured thematic spaces

space is a simplifying assumption that sidesteps some difficult issues in thematic integration. Formally, a granulation of  $U$  is a partition of  $U$  into disjoint subsets whose union is  $U$ .

To model the integration in this case, we must note that the integrated theme associated with a spatial location must contain the atomic elements of each of the constituent themes. So, the product construction gives:

$$T_1 \otimes T_2 = \{t_1 \cap t_2 : t_1 \in T_1, t_2 \in T_2, t_1 \cap t_2 \neq \emptyset\}$$

$$p_1 : t_1 \cap t_2 \mapsto t_1, \quad p_2 : t_1 \cap t_2 \mapsto t_2$$

$$f_1 \otimes f_2 : s \mapsto sf_1 \cap sf_2$$

The condition that  $t_1 \cap t_2 \neq \emptyset$  is interesting, in that it constrains the possibilities for integration into a combined spatio-thematic layer. This leads to a major difference between our first two cases. In the first case, the integrated layer is always guaranteed to exist, but in the second case, the integrated layer will only exist if the two constituent layers are consistent, in a sense now to be defined. Two layers  $f_1 : S \rightarrow T_1$  and  $f_2 : S \rightarrow T_2$ , to be *consistent* if and only if for all locations  $s \in S$ ,  $sf_1 \cap sf_2 \neq \emptyset$ . It is not difficult to show that if two layers  $f_1 : S \rightarrow T_1$  and  $f_2 : S \rightarrow T_2$  are consistent, then the product construction given above is well-defined and yields a unique integrated layer that ensures that the arrow diagram in Fig. 2 commutes.

For example, consider the following collection of land cover types: conifer woodland (C), broad-leaved woodland (B), natural grassland (N), moorland (M), heathland (H), orchard (O), pasture (P), and arable land (A). (While this typology is only an example, many land cover data sets, such as CORINE discussed in

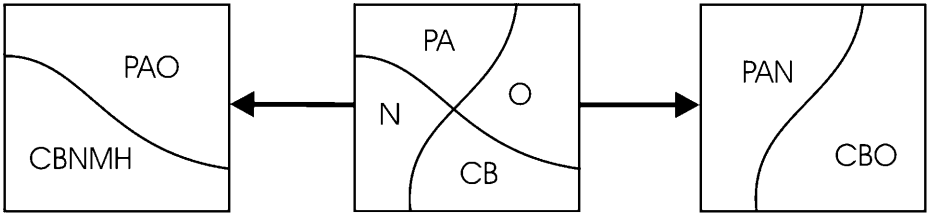


Fig. 5. The product of two layers with partition-based thematic spaces

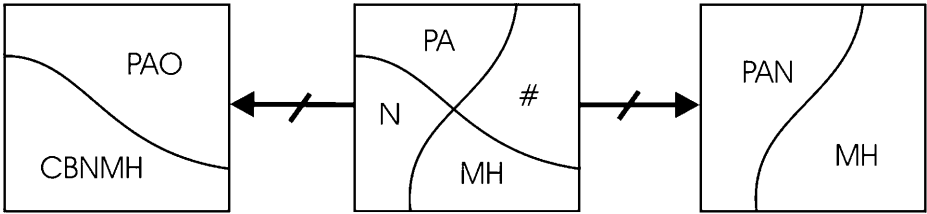


Fig. 6. Inconsistent layers with partition-based thematic spaces

section 4, contain similar classes and structure.) So,  $U = \{C, B, N, M, H, O, P, A\}$ . Thematic space  $T_1$  consists of the partition into agricultural (orchard, pasture, arable land) and forest/semi-natural types (conifer woodland, broad-leaved woodland, natural grassland, moorland, heathland). Thematic space  $T_2$  consists of the partition into herbaceous plant cover (natural grassland, pasture, arable land), shrubs (moorland, heathland), and trees (conifer woodland, broad-leaved woodland, orchard). So:

$$T_1 = \{\{P, A, O\}, \{C, B, N, M, H\}\}$$

$$T_2 = \{\{P, A, N\}, \{M, H\}, \{C, B, O\}\}$$

In this case, the integrated thematic space, consisting of all non-empty intersections of blocks in  $T_1$  and  $T_2$ , is:

$$T_1 \otimes T_2 = \{\{P, A\}, \{O\}, \{N\}, \{M, H\}, \{C, B\}\}$$

Taking the spatial framework as in Fig. 3, Fig. 5 shows two thematic layers and their product, while Fig. 6 shows two inconsistent layers. The inconsistency, represented by “#” results from classifications of the same location as agricultural and shrubs, and at least in this typology, such classifications are inconsistent.

### 2.3 Case C: Hierarchical Structure in the Thematic Spaces

The partition-based thematic structure in Case B is essentially “flat,” with all blocks of the partition at the same level, and two blocks either being disjoint or

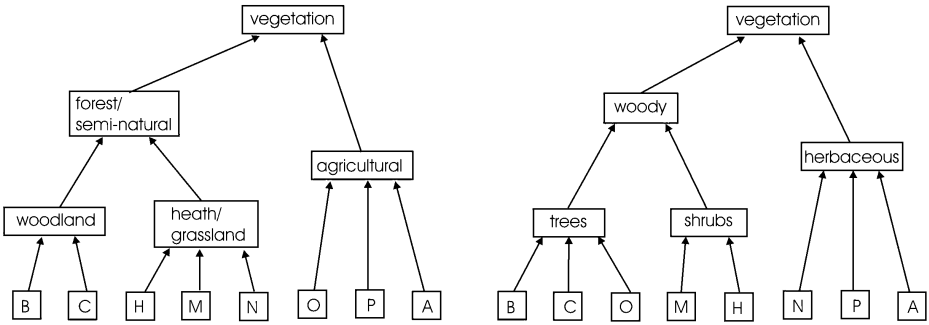


Fig. 7. Two hierarchical thematic spaces

equal. In Case C, we provide some hierarchical structure to the themes, without allowing the full generality of a partially ordered set. As with the other cases, we assume a set of “atomic” themes, which will form the basis of the hierarchy. Formally, assume throughout a set  $U$  of themes, called *atoms*. The power set  $\wp(U)$  is the set of all subsets of  $U$ .

**Definition 1.**  $H$  is a  $U$ -hierarchy if it satisfies the following conditions:

1.  $H$  is a labeled subset of  $\wp(U)$
2.  $H$  contains all the singleton sets  $\{u\}$  for  $u \in U$ , with each  $\{u\}$  labeled by  $u$ .
3.  $H$  does not contain the empty set.

A hierarchy induces a structure on its labels, where labels are the names of themes in our applications. Let  $T$  be the set of labels of  $H$  and, for each  $t \in T$ , let  $t\alpha$  be the set of atoms in  $U$  that is labeled by  $t$ . (Note that by property 2 above,  $u\alpha = \{u\}$ ). A partial ordering  $\leq$  of labels is induced on  $T$ , where  $t \leq t'$  if  $t\alpha \subseteq t'\alpha$ . The intuition is that  $t \leq t'$  if and only if the type corresponding to  $t'$  subsumes the type corresponding to  $t$ . Note that  $T$  is not necessarily a lattice, as it may not be closed with respect to lattice join and meet. However, we may define the lattice meet operation as follows:

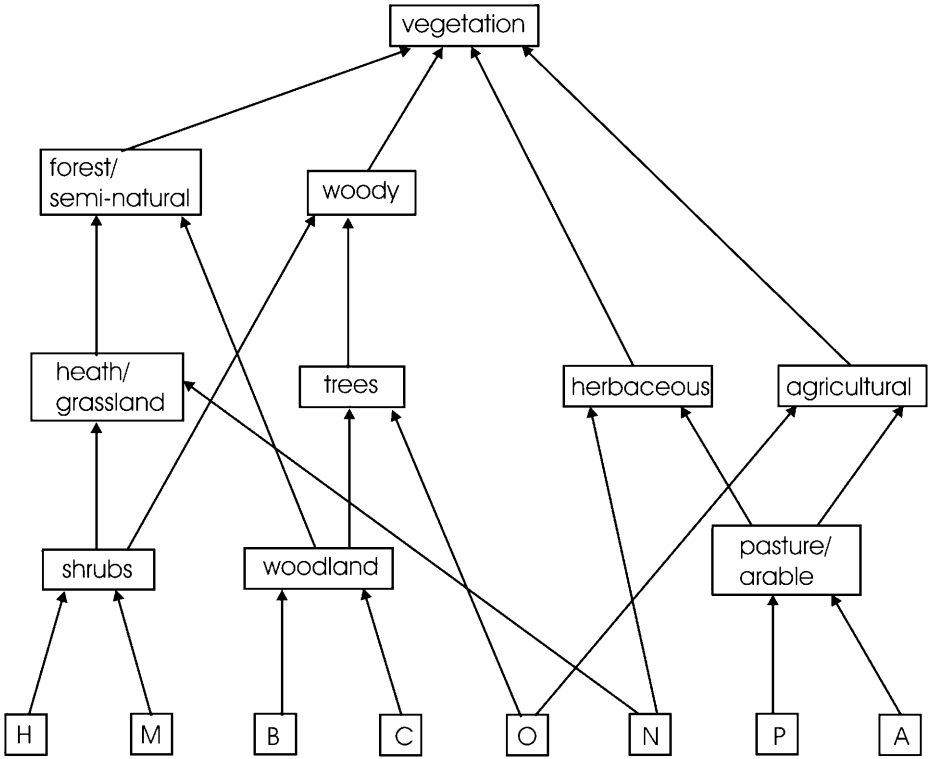
$$t \wedge t' = t'' \iff t\alpha \cap t'\alpha = t''\alpha$$

If later on, we construct a non-bottom meet of themes that does not exist in the hierarchy, then we will need to construct a new label for the new theme.

Fig. 7 shows two thematic hierarchies that we will use as an example of the constructions developed in this section. The single letter symbols in the bottom level of the hierarchy refer to the land cover classes used in the example for Case B, and the thematic labels are shown rather than the underlying subsets.

We are now ready to make the construction that formalizes the integration of two thematic hierarchies. Given two thematic hierarchies  $T_1$  and  $T_2$  on the same set  $U$  of atoms, make the following construction:

$$T_1 \odot T_2 = \{t_1 \wedge t_2 : t_1 \in T_1, t_2 \in T_2, t_1 \wedge t_2 \neq \perp\}$$



**Fig. 8.** The product of the hierarchical thematic spaces in Fig. 7

Fig. 8 shows the integration of the two thematic hierarchies in Fig. 7. Note the existence of a newly constructed theme, “pasture/arable,” needed because the meet of themes “herbaceous” and “agricultural” is not in the original hierarchies as a label.

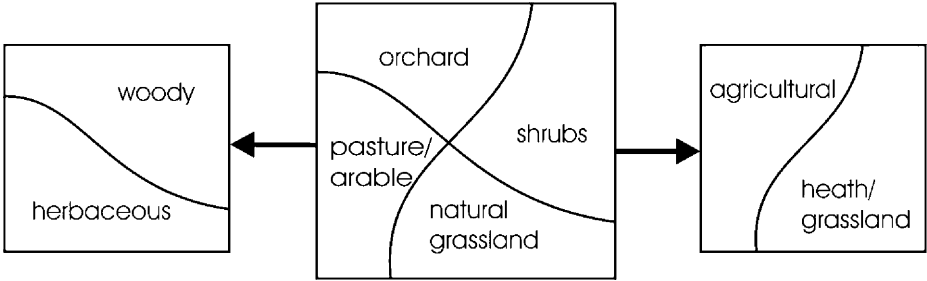
A different symbol  $\odot$  has been used for the integration construction, because it is not necessarily a product. The projection functions

$$p_1 : t_1 \wedge t_2 \mapsto t_1, \quad p_2 : t_1 \wedge t_2 \mapsto t_2$$

are not in general well-defined. Problems occur when there is multiple inheritance in the constituent hierarchy, that is when a node has more than one direct ancestor. In cases when each node has a unique ancestor, the projection functions are well-defined, and in that case we have the product construction, as in the other cases.

The next step is to construct the integration layer. Firstly, as in Case B, we note that this layer may not always exist. Define two layers  $f_1 : S \rightarrow T_1$  and  $f_2 : S \rightarrow T_2$ , to be *consistent* if and only if for all locations  $s \in S$ ,  $s f_1 \wedge s f_2 \neq \perp$ .

Assume given two consistent layers,  $f_1 : S \rightarrow T_1$  and  $f_2 : S \rightarrow T_2$ . We have already shown how to construct the integrated theme space  $T_1 \odot T_2$ . The integrated layer function is constructed as follows:



**Fig. 9.** The product of two layers with hierarchically structured thematic spaces

$$f_1 \odot f_2 : s \mapsto sf_1 \wedge sf_2$$

As in case B, it is not difficult to show that if two layers  $f_1 : S \rightarrow T_1$  and a  $f_2 : S \rightarrow T_2$  are consistent, and the two thematic hierarchies  $T_1$  and  $T_2$  have only single inheritance, then the product construction is well-defined and yields a unique integrated layer that ensures that the arrow diagram in Fig. 2 commutes.

Fig. 9 shows an example of the integration of two consistent layers, where the thematic hierarchies and their meet are given in Fig. 7 and 8.

### 3 Granularity Issues

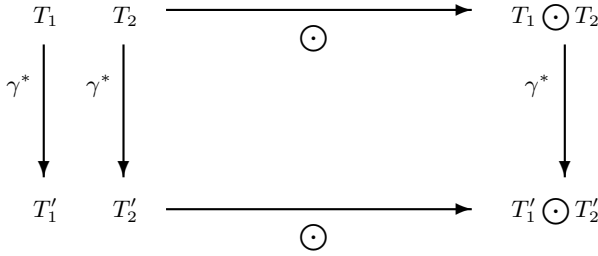
Previous work has provided a formal framework for spatial and thematic resolution separately [10, 11]. For a multi-resolution model, an understanding of the relationship between integration and coarsening is required. Ideally, we would like these operations to commute, so that if we coarsen two layers and then integrate them, we will achieve the same as if we integrated first and then coarsened the merged layer. This is achieved when the arrow diagram in Fig. 10 commutes. We examine this issue in the context of Case C, with respect to thematic granularity. In Case C, suppose we have two thematic hierarchies  $T_1$  and  $T_2$  on the same set  $U$  of atoms. A granulation of  $U$  can be expressed as a (many-to-one) function  $\gamma : U \rightarrow U'$ . This granulation induces a coarsening of each hierarchy, using the construction:

$$\gamma^* : T_1 \rightarrow T'_1$$

$$\gamma^* : t \mapsto t' \text{ where } t' \text{ is the unique element such that } \alpha t' = t\alpha\gamma$$

For example, suppose we coarsen our land cover classes as follows:  $\gamma : B \mapsto W, C \mapsto V, H \mapsto V, M \mapsto X, N \mapsto X, O \mapsto Y, P \mapsto Y, A \mapsto Z$ . Then apart from atoms and the top class, the hierarchy on the left of fig 7 has classes (given in extensional form)  $\{W, V\}$ ,  $\{V, X\}$ ,  $\{Y, Z\}$ , and  $\{W, V, X\}$ , while the hierarchy on the right has classes (given in extensional form)  $\{W, V, Y\}$ ,  $\{V, X\}$ ,  $\{X, Y, Z\}$ , and  $\{W, V, X, Y\}$ . Integrating these hierarchies gives a hierarchy with classes,  $\{W, V\}$ ,  $\{V, X\}$ ,  $\{Y, Z\}$ ,  $\{W, V, X\}$ ,  $\{W, V, Y\}$ ,  $\{X, Y, Z\}$ , and  $\{W, V, X, Y\}$ . This





**Fig. 10.** Coarsening and integration working together

is the same result as integrating first, as in Fig. 9, and then coarsening. However, in general a weaker result holds between an integrated coarsening and a coarsened integral.

We can see this in the case of the partitions introduced in Case B. The set of partitions of an underlying thematic space  $U$  may be made into a poset by introducing the following partial order relation. For partitions  $T_1$  and  $T_2$ , define  $T_1 \leq T_2$  if and only if  $\forall x \in T_1 \exists y \in T_2. x \subseteq y$ . The set of partitions of  $U$  may be made into a lattice by defining meet and join operations. For partitions  $T_1$  and  $T_2$ ,  $T_1 \wedge T_2 = \{x \cap y | x \in T_1, y \in T_2, x \cap y \neq \emptyset\}$ , and there is a similar but slightly more complex construction for the join of two partitions. Further details may be found in [11].

A thematic coarsening operation on a partition may be viewed as a many-one function on the underlying thematic space. Let  $c : U \rightarrow V$  be a thematic coarsening operation. Function  $c$  induces a partition  $\Pi_c$  on  $U$ , where two elements belong to the same block of  $\Pi_c$  if they have the same image under  $c$ . For any partition  $T$  of  $U$ , the coarsening  $c$  of  $U$  induces the coarsening  $\Pi_c \vee T$  of  $T$ . For the diagram in Fig. 10 to commute, we must have:

$$\Pi_c \vee (T_1 \odot T_2) = (\Pi_c \vee T_1) \odot (\Pi_c \vee T_2)$$

But,  $T_1 \odot T_2$  is just  $T_1 \wedge T_2$  in the lattice of partitions of  $U$ , so we have the following:

$$\Pi_c \vee (T_1 \wedge T_2) = (\Pi_c \vee T_1) \wedge (\Pi_c \vee T_2)$$

as a condition for a commutative diagram in in Fig. 10. Now, this is just the distributive property of lattices, and it is easy to find examples of partition spaces that form non-distributive lattices. However, the weaker property:

$$\Pi_c \vee (T_1 \wedge T_2) \leq (\Pi_c \vee T_1) \wedge (\Pi_c \vee T_2)$$

does hold. This expresses formally the intuition that it is better to integrate first then coarsen, rather than the other way round.

## 4 Case Study

The CORINE project, started in 1985, provides a framework for coordinating the collection of consistent land cover data across the European Community. Data sets concerning the 44 different CORINE land cover classes now exist for more than 20 European countries, including Britain. A CORINE data set maps land cover units with a minimum spatial granularity of 25ha based on 44 land cover classes arranged in a 3-tier taxonomy [12]. CORINE data is commonly used within a variety environmental applications, such as environmental impact assessment, monitoring environmental change and management of biodiversity.

By updating legacy cartographic mapping products, the Ordnance Survey of Great Britain (OSGB) has recently developed the Digital National Framework (DNF), which aims to provide a consistent national framework for digital mapping in GB. Marketed as MasterMap, the “definitive” digital map database, the DNF comprises large scale general purpose geographic data [13]. DNF data is specifically designed to be integrated with other digital data using unique persistent digital identification numbers, called topographic object identifiers (TOIDs). Since DNF data is derived from older cartographic data, the feature classifications, including land cover classifications, are essentially those described by Harley [14].

The two data sets, CORINE and DNF data, provide an interesting contrast. The former is a relatively specialized land cover map, whilst the latter is a general purpose mapping product aimed specifically at integration with other spatial data products. CORINE provides detailed thematic information based on a hierarchy of land cover classes, while DNF data uses a relatively coarse land cover taxonomy. At the same time, the large scale DNF data provide much greater spatial detail than is available in CORINE data. The two data sets provide different, but potentially complementary information. The ability to successfully integrate such data set could, therefore, be beneficial in a variety of environmental applications: indeed the automated integration of CORINE data with other land cover data sets has already been addressed in the literature (see, for example [15]).

To illustrate the usefulness of the layer-based approach for integrating spatio-thematic information, the remainder of this section describes the integration of CORINE and DNF data for a small region in GB using prototype software that implements the layer integration techniques described in previous sections.

### 4.1 Software Prototype

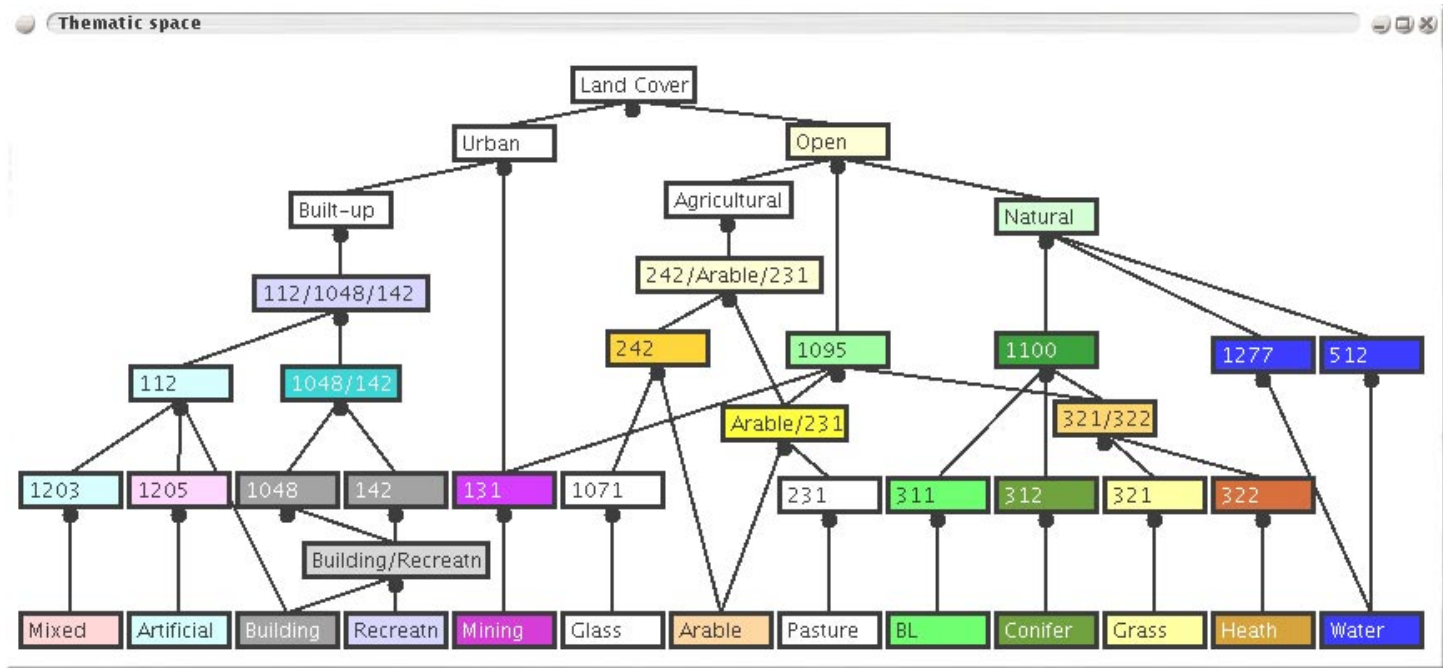
Using Java, a prototype data integration application was developed and tested with example data from the CORINE and DNF land cover data sets. The prototype enables heterogeneous data to be integrated using the layer-based approach as described in section 2.3. The user interface for the prototype consists of two linked windows allowing manipulation of the spatial and thematic aspects of the data respectively. The spatial window, illustrated in Fig. 11 with example

DNF data, provides users with limited spatial manipulation capabilities, such as zooming, panning, and simple spatial queries.



**Fig. 11.** Spatial window with example DNF data

The thematic window, illustrated in Fig. 12, enables users to build a “thematic map” of the relationship between the classifications used in different data sets. Performing the actual data integration is effectively a process of defining a suitable set of thematic atoms, and specifying the relationships between those atoms and the thematic classes in each data set, in this case CORINE and DNF. The metadata and documentation available for CORINE and DNF define the hierarchy of different thematic classes used in each data set, as well as providing adequate descriptions of the different classes to help in deciding on suitable atoms (see [12, 13]). The prototype software assists users in building the thematic map in several ways, described in more detail in section 4.2: by automatically restructuring thematic maps; by making thematic maps persistent; by providing dynamic consistency maps (identification of inconsistencies); as well as performing the actual data integration (resolution of inconsistencies). It is important to note that the process of actually relating atoms and thematic classes in different data sets, for example identifying synonymous and homonymous land use classes, remains primarily a human activity, although future work aims to automate more of this process.



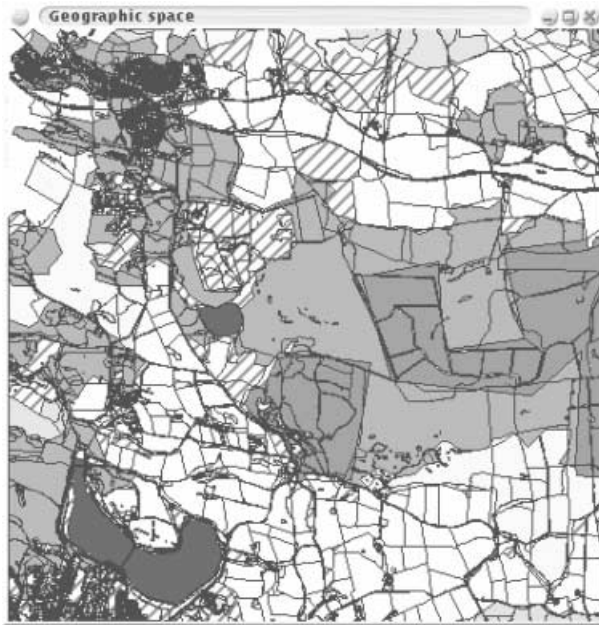
**Fig. 12.** Thematic window with combined CORINE/DNF hierarchy. Numbers indicate CORINE land cover class code; labels indicate DNF land cover class

## 4.2 Software Prototype Features

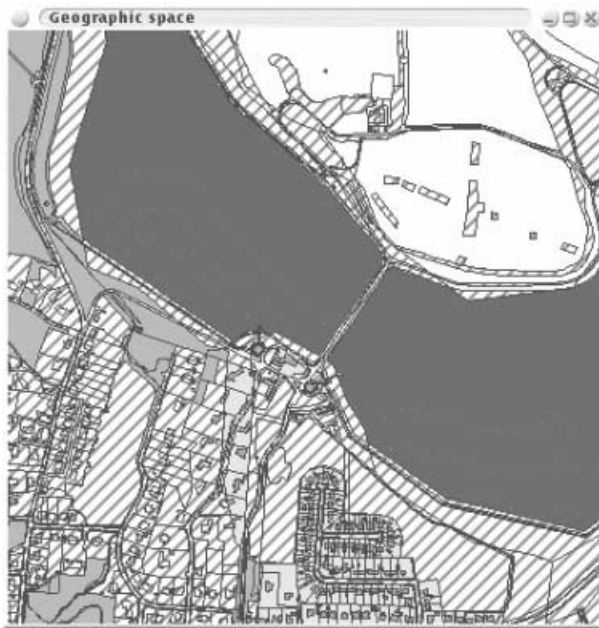
The software can help with structuring the thematic space by automatically inserting new thematic classes where necessary within the hierarchy to ensure every non-bottom meet of themes exists in the hierarchy (as done with the “pasture/arable” class in section 2.3). The software also allows a thematic map to be saved in a persistent format, in a database or data file. The thematic map in Fig. 12 is not derived from the complete set of classes for DNF and CORINE data, only those classes that are represented within the spatial extents of the example data sets. However, it would be possible, to share a general purpose thematic map between different specific applications. For example, were a complete thematic map for the relationship between DNF and CORINE data to be constructed, it might subsequently be used for integrating any DNF and CORINE data automatically.

The software can be used for both identification and resolution of inconsistencies. Before completing data integration, the prototype enables users to produce an intermediate “consistency map,” identifying the locations of inconsistencies (as defined in section 2.3, those locations whose themes have no meet). Fig. 13 shows a consistency map for the partially completed integration of the example CORINE and DNF data sets. The hatched areas are not consistent. The consistency map is useful as it gives an impression of the spatial extents and distributions of inconsistencies. While the two data sets are broadly consistent for the predominately rural areas shown in Fig. 13 (relatively few hatched areas), some regions of inconsistency persist. Zooming in on some of the detail in the bottom left hand corner of the map shows that this predominately urban area has much higher levels of inconsistency, as shown in Fig. 14, indicating either a problem with the structure of the thematic map or an underlying misclassification in one or both of the data sets.

Using such intermediate consistency maps to identify inconsistencies can help inform users as to amendments to the structure of the thematic hierarchy that may be needed. When users are satisfied with the structure of their thematic map, the data integration proceeds for consistent locations in the normal way, by finding the meet of their themes. Inevitably, some inconsistencies will always remain. In this prototype, a very simple mechanism is used to resolve such inconsistencies. If the themes associated with two locations have no meet, the join of themes is used instead. The rationale for this is that the join represents a coarsening of two inconsistent themes to their consistent least upper bound. For example, while DNF class 1205 (artificial impermeable surfaces) and CORINE class 142 (sports and leisure facilities) have no meet, the join of these classes (built-up area) can be used as a coarser consistent theme where inconsistencies arise between these two classes. Using the join of two themes is a simple but effective mechanism for consistency resolution. Other work by the authors is currently investigating more sophisticated rule-based mechanisms for resolving these inconsistencies when they arise.



**Fig. 13.** Consistency map (hatching indicates inconsistency)



**Fig. 14.** Consistency map detail: inconsistencies in urban area (hatching indicates inconsistency)

## 5 Conclusions

This paper constructs a formal framework in which the integration of structured thematic layers may be discussed. The approach taken is to think of integration as essentially a product operation, and thereby incorporate some formal machinery on product constructions from universal algebra. The thematic spaces considered are given increasing amounts of structure, from straight sets to blocks in a partition to taxonomic hierarchies. We show that in these cases the product construction does have something useful to contribute, and note how basic problems of semantic mismatch have a direct correspondence to breakdowns in the structured products. The work is explained using some small examples, and then applied to a significant issue of relating CORINE and DNF thematic data types. The issue of the interplay between changing resolution (e.g., thematic coarsening) and resolving inconsistencies is also discussed, but the details of that remain to be worked out more fully in a later paper.

Whereas most work in this field concentrates on issues of spatial integration, this paper has redressed the balance by focusing on the thematic aspects of the problem. A fuller theory would develop both sides in tandem. Another issue that is sidestepped in this work is the question of relationships between “atoms” in cases B and C. We have assumed a common set of atoms, and heterogeneity provided by different groupings (partitions or hierarchies) of the atoms. In general the issue of relating atoms from different thematic classification is a difficult one, and the subject of further work.

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## References

1. Date, C.: *Introduction to Database Systems*. 7 edn. Addison-Wesley (2000)
2. Stonebraker, M.: *Readings in Database Systems*. Morgan Kaufmann, CA (1984)
3. Sowa, J.: *Building, Sharing and Merging Ontologies*. <http://www.jfsowa.com/ontology/ontoshar.htm> (2001)
4. Kashyap, V. and Sheth, A.: Semantic Heterogeneity in Global Information Systems: The Role of Metadata, Context and Ontologies. In: Papazoglou, M.P. and Schlageter, G., eds.: *Cooperative Information Systems*. Academic Press, San Diego (1998) 139–178
5. Fonseca, F., Egenhofer, M., Agouris, P., Câmara, G.: Using Ontologies for Integrated Geographic Information Systems. *Transactions in Geographic Information Systems* **6** (2002) 231–257

6. Uitermark, H., van Oosterom, P., Mars, N., and Molenaar, M.: Ontology-Based Spatial Data Set Integration. In: *Proceedings of STDBM'99, Workshop on Spatio-Temporal Database Management*, Edinburgh, Scotland. (1999)
7. Guarino, N.: Semantic Matching: Formal Ontological Distinctions for Information Organization, Extraction, and Integration. In: Pazienza, M., ed.: *Proceedings International Summer School, SCIE-97*, Frascati, Italy. Volume 1299 of Lecture Notes in Computer Science., Berlin: Springer (1997) 139–170
8. Berry, J.: Fundamental Operations in Computer Assisted Map Analysis. *International Journal of Geographical Information Systems* **1** (1987) 119–136
9. Tomlin, C.: Cartographic Modelling. In Maguire, D., Goodchild, M., Rhind, D., eds.: *Geographical Information Systems*. Volume 1. 1st edn. Longman, England (1991) 361–374
10. Worboys, M.: Computation with Imprecise Geospatial Data. *Journal of Computers, Environment and Urban Systems* **22** (1998) 85–106
11. Worboys, M.: Imprecision in Finite Resolution Spatial Data. *Geoinformatica* **2** (1998) 257–280
12. Bossard, M., Feranec, J., and Otahel, J.: CORINE Land Cover Technical Guide. Technical Report 40, European Environment Agency, Copenhagen (2000)
13. Ordnance Survey of Great Britain: Digital National Framework. Consultation Paper 1, Ordnance Survey of Great Britain, Southampton, UK (2000)
14. Harley, J.: Ordnance Survey Maps: A Descriptive Manual. Technical Report, Ordnance Survey of Great Britain, Southampton, UK (1975)
15. Fuller, R. and Brown, N.: A CORINE Map of Great Britain by Automated Means, Techniques for Automatic Generalization of the Land Cover Map of Great Britain. *International Journal of Geographical Information Systems* **10** (1996) 937–953